

A microscopically derived formula for alpha-decay half-lives

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Abstract Although Geiger-Nuttall (GN) law gives a single straight line, if we consider the experimental data of alpha particle emitters including heavy and super heavy nuclei with proton numbers as large as 118, instead of getting a single linear path we observe several linear segments with different slopes and intercepts. This problem is overcome when the experimental results in logarithm form are plotted as a function of Viola-Seaborg (VS) parameters with values of parameters set by hand. By using the fundamental principle of decay, we derive a formula of logarithm of half-lives in terms of well-defined parameters or coefficients and this replaces the empirical VS rule.

Keywords Alpha decay, Q-value, resonance, decay half-life, Analytical formula

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1 Introduction

The quintessential α -radioactivity has been studied by many physicists so far and has opened doors for laying a rigid foundation and development of nuclear physics [1,2,3,4]. Gamow was the first and foremost to apply quantum mechanics to a nuclear physics problem by providing the first model to explain α -decay and propounded that the process involves tunneling of an α -particle through a large barrier [5]. A profound knowledge of this quantum mechanical effect enables one to obtain the Geiger-Nuttall law which relates the decay constant of a radioactive isotope with the energy of the α particles emitted.

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WKB approximations have been applied to study α -decay rate of many elements. Lifetimes of several heavy elements with $Z = 102 - 120$ have been estimated by theoretically calculating the quantum mechanical tunneling probability in a WKB framework and also using the *DDM3Y* effective nuclear interaction whose results have shown good agreement over a wide range of experimental datas [6, 7, 8, 9, 10, 11].

Our acquaintance with the well known Geiger-Nuttall (GN) law is age old. In fact, the formulation of the Geiger-Nuttall (GN) law in 1911 was a landmark in itself. The GN law states that the α -decay half life $T_{1/2}$ is related to the energy of α -decay process (α -decay value) Q_α as $\log_{10}T_{1/2} = A(Z)Q_\alpha^{-1/2} + B(Z)$, where $A(Z)$ and $B(Z)$ are the coefficients which are determined by fitting experimental data [12, 13, 14]. The GN law holds good for a restricted experimental data sets available but is invalid in general. In retrospect, we can see that although GN law gives a single straight line but if we consider the experimental data of α -particle emitters including heavy and super heavy nuclei with proton numbers as large as 118, instead of getting a single linear path we observe several linear segments with different slopes and intercepts. This problem is overcome when the experimental results in logarithm form are plotted as a function of Viola-Seaborg (VS) parameter, $\tilde{V} = \frac{a'Z_D + b'}{\sqrt{Q_\alpha}} + c'Z_D$, where a' , b' and c' are constants [15]. The striking feature of empirical Viola-Seaborg (VS) rule i.e $\log_{10}T_{1/2} = \frac{a'Z_D + b'}{\sqrt{Q_\alpha}} + c'Z_D + d'$ with $a' = 1.478$, $b' = 8.714$, $c' = -0.183$ and $d' = -34.699$, is its superiority over GN law as it satisfactorily results in a straight line for a wide range of experimental data. As stated before the VS rule is precise but is an empirical one. Thus we strive for a formula of logarithm of half-lives in terms of well-defined parameters or coefficients so that the empirical nature of VS rule is apparently obscured.

Qi et al. have mentioned the validity and generalization of the GN law along with its microscopic basis. Also they have incorporated the tunneling process in the R-matrix theory and put forth an extended form of GN law [16, 17, 18].

To fully understand the R-matrix theory for the decay of a cluster or a particle, we look from the perspective of S-matrix theory of resonance scattering or the transition scattering from an isolated quasi-bound state to a scattering state as detailed in [19]. For completeness, we can highlight the method as follows:

In S-matrix method, resonance is considered as a pole in the complex energy plane. Adding to that, the real part of the pole signifies the resonance energy or the Q-value of decay and the imaginary part represents the width which in turn gives the decay half-life of the system constituting of an α -cluster and the residual nucleus [20].

In the context of transition from quasi-bound state to a scattering state, we can write the width in terms of wave function at resonance and Coulomb functions in two ways:

i) By matching the normalized regular solution $u(r)$ of the modified Schrödinger equation and the distorted outgoing Coulomb function at a distance $r = R$ is

$$u(R) = N_0 |G_0(\eta, kR) + iF_0(\eta, kR)|, \quad (1)$$

where F_0 and G_0 are the regular and irregular Coulomb functions. Thus, the decay width [21] is expressed as

$$\Gamma = \frac{\hbar^2 k}{\mu} |N_0|. \quad (2)$$

ii) We express the general formula of the α -decay width [22] as follows:

$$\Gamma = 2\pi |\langle \psi | H - H_0 | \phi \rangle|^2, \quad (3)$$

where ψ is a bound initial state for the decaying nucleus and ϕ is a final scattering state for the α +daughter system. The Hamiltonians H_0 and H are associated with ϕ and ψ , respectively.

Both the expressions are same in one way or another. Precisely in the first expression Γ depends on a radial distance R which is represented crudely. This results in uncertainty in measuring certain data. It is interesting at this point to consider the second formula which is more effective in deriving an analytical expression for half-life in terms of the resonant wave function of an exactly solvable potential and the regular Coulomb function. Ultimately, by applying approximations on the functions of the expression of half-life, we derive a condensed formula for the logarithm of half-life in terms of the decay energy and mass and charge numbers of the α -emitter. It is noteworthy that the found out condensed formula bear close resemblance with the VS rule. Thus we can say that finding half-lives from a derived formula is the paramount of this paper.

In section II we describe in detail the formulation and derivation of half-lives. In section III the formula is being implemented. Section IV includes the conclusion of the sought problem.

2 Theoretical framework

2.1 Decay width or half-life of α -decay

We remark that the α -decay process where the α -cluster in the decaying nucleus is controlled by an attractive nuclear potential, $V_N(r)$ and the α -particle outside the nucleus by the point-charge Coulomb potential i.e $V_C^P = \frac{Z_1 Z_2 e^2}{r}$.

In a simple picture we represent $H - H_0$ as the difference between the potentials in the two cases viz. the nuclear potential and the point-charge Coulomb potential i.e

$$H - H_0 = \{V_N(r) + V_C(r)\} - V_C^P(r) = V_{eff}(r) - V_C^P(r), \quad (4)$$

where $V_C(r)$ is the Coulomb potential given by

$$V_C(r) = \begin{cases} 0 & \text{if } r > R_0, \\ \frac{Z_1 Z_2 e^2}{2R_C} [3 - (\frac{r}{R_C})^2] & \text{if } r \leq R_0, \end{cases} \quad (5)$$

In our approach, we calculate the decay width by taking into account the α -decay process where there is transition of an α -cluster from an isolated quasi-bound state to a scattering state. The initial system is related with the instability with the quasi-bound state of the decaying nucleus. Along with that the final state is the scattering state of the α -daughter system.

Now, we solve the Schrödinger equation using the effective potential which is the amalgamation of the nuclear potential and the electrostatic potential to get the radial part of the initial and final state of the wave function.

The radial part of the initial state wave function is

$$\psi_{nl}(r) = \frac{u_{nl}}{r}. \quad (6)$$

Additionally, the final state wave function can be written considering the motion of the α -particle relative to the daughter nucleus as a scattering state wave function corresponding to the α -particle in point charge Coulomb potential [23]:

$$\phi(r) = \sqrt{\frac{2\mu}{\pi\hbar^2 k}} \frac{F_l}{r}. \quad (7)$$

where $k = \sqrt{2\mu E_{c.m.}/\hbar}$, $E_{c.m.}$ stands for the center-of-mass energy, $\mu = m_n \frac{A_\alpha A_D}{A_\alpha + A_D}$ is the reduced mass of the system with m_n giving the mass of a nucleon, A_α represent the mass number of α particle, A_D represent the mass number of the daughter nucleus and F_l is the regular Coulomb wave function for a given partial wave l . The factor $\sqrt{\frac{2\mu}{\pi\hbar^2 k}}$ is a normalization factor of the scattering wave function.

The mean-field approximation [24] is applied for the nucleon-nucleon interaction. Moreover, the process of double folding of the α +nucleus potential $V_N(r)$ along with the electrostatic term, we get a parabola at a close radial distance r . Again simulation of the effective potential as a function of distance is done and is solved exactly in the Schrödinger equation. Consequently the solution of which is the wave function $u_{nl}(r)$ for $l = 0$ in the interior region.

Based on the Gell-Mann-Goldberger transformation [21], the expression for the decay width becomes

$$\Gamma = \frac{4\mu}{\hbar^2 k} \frac{|\int_0^R F_l[V_{eff}(r) - V_C^P(r)]u_{nl}(r)dr|^2}{\int_0^R |u_{nl}(r)|^2 dr}. \quad (8)$$

For the normalization of the interior wave function the factor $\int_0^R |u_{nl}(r)|^2 dr$ is used. The resonant wave function $u_{nl}(r)$ decreases rapidly with distance outside the Coulomb barrier radius r_B . For this reason, we apply the box normalization condition for the wave function $\int_0^R |u_{nl}(r)|^2 dr = 1$ for $R \approx r_B$.

Also, we are quite familiar with the relation between decay half-life $T_{1/2}$ and the width:

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma}. \quad (9)$$

By using (9), we get a new expression of $T_{1/2}$

$$T_{1/2} = \frac{0.693 \hbar^3 k}{4\mu} \frac{1}{J}, \quad (10)$$

$$J = \left| \int_0^R F_l [V_{eff}(r) - V_C^P(r)] u_{nl}(r) dr \right|^2. \quad (11)$$

Now, the regular Coulomb wave function $F_l(r)$ can be expressed as [25]

$$F_l = A_l \rho^{l+1} f_l(\rho), \quad (12)$$

where $\rho = kr$, Sommerfeld parameter $\eta = \frac{\mu}{\hbar^2} \frac{Z_\alpha Z_D e^2}{k}$,

$$f_l(\rho) = \int_0^\infty (1 - \tanh^2 \epsilon)^{l+1} \cos(\rho \tanh \epsilon - 2\eta \epsilon) d\epsilon, \quad (13)$$

$$A_l = \frac{\sqrt{1 - \exp(-2\pi\eta)}}{2^l \{2\pi\eta(1 + \eta^2)(2^2 + \eta^2) \dots (l^2 + \eta^2)\}^{1/2}}. \quad (14)$$

In particular, for $l = 0$, A_l is given by

$$A_0 = \left\{ \frac{1 - \exp(-2\pi\eta)}{2\pi\eta} \right\}^{\frac{1}{2}}. \quad (15)$$

We make a lay out (Figs. 1(a)-1(d)) to properly describe the radial dependence of the three terms in the integrand $J_1 = \int_0^R F_l [V_{eff}(r) - V_C^P(r)] u_{nl}(r) dr$ mentioned in (11), the modulus of the resonance state wave function, $|u(r)|$, the regular Coulomb wave function, $F_0(r)$ for $l = 0$ and the combined nuclear and Coulomb potential, $V_{eff}(r)$ by taking the α +daughter system ($\alpha + {}^{220}_{88}\text{Ra}$) with Q-value of decay or energy $E = 7.298$ MeV representing the upper curve and the difference of potentials, $V_{eff}(r) - V_C^P(r)$ representing the lower curve of Fig. 1(c). In Fig. 1(d) the total integrand J_1 multiplied by a scaling factor of 10^{11} is shown. It is visible from the plot that the integrand shows a peak in the region close to the Coulomb barrier radius, $r_B = 10.11$ fm. The plots indicate that the wave function of the resonance state decreases exponentially in the barrier region. Also the potential difference ie. $(V_{eff} - V_C^P)$ becomes zero in the region $r \geq r_B$. But the Coulomb function is very small at small values of r . We also find that, although the integrand J is dependent on distance $r=R$ but it is independent of the distance R in the region $r \geq r_B$. Thus the value of decay time $T_{1/2}$ which is dependent on J is rather independent of R in the region $r \geq r_B$.

2.2 The effective α +nucleus potential

We use a potential [26] which is a function of radial variable r and can be represented in the form as follows:

$$V_{eff}(r) = \begin{cases} V_0[S_1 + (S_2 - S_1)\rho_1] & \text{if } r \leq R_0, \\ V_0 S_2 \rho_2 & \text{if } r \geq R_0, \end{cases} \quad (16)$$

where V_0 is the strength of the potential with value 1 MeV.

$$\rho_n = \frac{1}{\cosh^2 \frac{R_0 - r}{d_n}}; n = 1, 2, \quad (17)$$

d_n accounts for the flatness of the barrier, d_1 deciding the steepness of the interior side of the barrier whereas the exterior side is judged by d_2 . R_0 is the radial position having value; $R_0 = r_0(A_\alpha^{1/3} + A_D^{1/3}) + 2.72$, $r_0 = 0.97$ fm. Since we are considering the α +nucleus system, Z_α represent the proton number of α particle, Z_D represent the proton number of the daughter nucleus. Moreover S_1 and S_2 are the depth and height of the potential, respectively, having values;

$$S_1 = -78.75 + \frac{3Z_\alpha Z_D e^2}{2R_c}, \quad (18)$$

$$S_2 = \frac{Z_\alpha Z_D e^2}{R_0} \left(1 - \frac{a_g}{R_0}\right), \quad (19)$$

where R_c is the Coulomb radius parameter; $R_c = r_c(A_\alpha^{1/3} + A_D^{1/3})$, $a_g = 1.6$ fm, $r_c = 1.2$ fm, $e^2 = 1.43996$ MeV fm. r_c and a_g are the distance parameters.

2.3 Expression for decay half-life

As previously mentioned, we are considering the problem of α +nucleus system with a specific energy value Q_α value and radius $R = r_B$, the values of Sommerfeld parameter η and parameter $\rho = kR$ are such that $\eta\rho \leq 50$ and $\rho \approx 10$. In this context, we now use the power series expansion and write the Coulomb wave function $F_l(r)$ as

$$F_l^{ps}(r) = C_l \rho^{l+1} G_l, \quad (20)$$

$$(n+1)(n+2l+2)G_{n+1} = 2\eta\rho G_n - \rho^2 G_{n-1}, \quad (21)$$

$$G_0 = 1, G_1 = \frac{\eta\rho}{(l+1)}, G_l = \sum_j G_j, \quad (22)$$

$$C_l^2 = \frac{P_l(\eta)}{2\eta} \frac{C_0^2(\eta)}{(2l+1)}, \quad (23)$$

$$P_l(\eta) = \frac{2\eta(1+\eta^2)(4+\eta^2)\dots(l^2+\eta^2)2^{2l}}{(2l+1)[(2l)!]^2}. \quad (24)$$

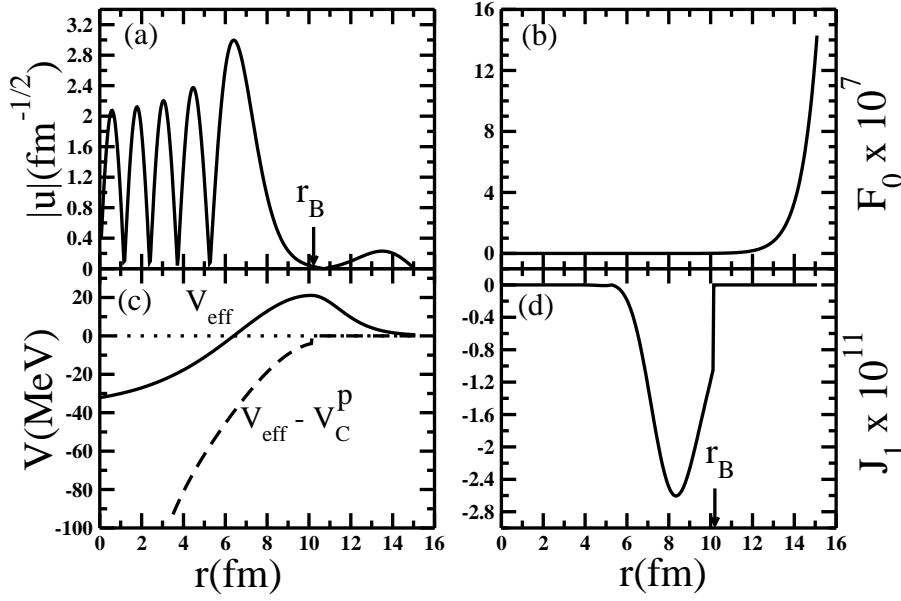


Fig. 1 Various terms for explaining the α -decay rate in s-wave of $\alpha + {}^{220}_{88}\text{Ra}$ system: (a) the modulus of the radial wave function at resonance, (b) the regular Coulomb wave function multiplied by 10^7 , (c) the α + daughter potential, V_{eff} representing the upper plot and $V_{eff} - V_C^P$ representing the lower plot in dotted line, (d) the integral $J_1 = \int_0^R F_l[V_{eff}(r) - V_C^P(r)]u_{nl}(r)dr$ multiplied by 10^{11} for the s-wave. The barrier radius $r_B = 10.11$ fm is shown in arrows.

For the case, $l = 0$, $P_0 = 2\eta$, and

$$C_0^2(\eta) = 2\pi\eta\{\exp(2\pi\eta) - 1\}^{-1}, \quad (25)$$

By putting the value of G_0

$$F_0^{ps}(r) = C_0\rho G_0, \quad (26)$$

We find that

$$F_l(r) = x_m F_l^{ps}(r), \quad (27)$$

where $x_m \approx 70$. Therefore, instead of computing function $F_l(r)$ using (12), we go by the simple power series expansion of F_l^{ps} using (20) multiplied by a factor $x_m = 70$. Fig. 1 clearly show that the magnitude of this function is zero near the origin $r = 0$ but increases predominantly at $r = r_B$ whereas the resonant wave function $u_{nl}(r)$ is very small beyond $r = r_B$. Hence, the integral J can be written in terms of $F_l(r) = x_m F_l^{ps}(r)$ at a point $r = R = r_B$ alongwith some multiplying factor which take care of the other contributions within the region $0 < r < R$.

The integral J now changes to

$$J = |c_f F_l(R)|^2 = |c_f x_m F_l^{ps}(R)|^2. \quad (28)$$

For a typical α +nucleus system, the value of $c_f(=\sqrt{J}/|x_m F_l^{ps}(R)|)$ is found to be 0.8 for $l = 0$. Using the above found J value, the decay half-life $T_{1/2}$ becomes

$$T_{1/2} = 0.693 \hbar q_l \frac{\exp(2\pi\eta)}{f_{ml}^2}, \quad (29)$$

where

$$q_l = \frac{2\eta(2l+1)}{P_l(\eta)\rho^{2l}}, \quad (30)$$

$$f_{ml} = \sqrt{4\mu/\hbar^2 k} \sqrt{2\pi\eta} c_f x_m \rho G_l, \quad (31)$$

We take the logarithm of both sides,

$$\log T_{1/2} = a\chi + c + d + b_l, \quad (32)$$

$$a = 1.4398\pi\sqrt{2(931.5)}/197.329 \quad (33)$$

$$\chi = Z_\alpha Z_D \sqrt{\frac{A_\alpha A_D}{(A_\alpha + A_D)Q_\alpha}}, \quad (34)$$

$$c = -2\log S, \quad (35)$$

$$d = -2\log D, \quad (36)$$

$$b_l = \log(q_l), \quad (37)$$

$$S = c_f x_m R G_l \frac{A_\alpha A_D \sqrt{Z_\alpha Z_D}}{A_\alpha + A_D}, \quad (38)$$

$$D = \frac{2 \times 931.5 \times \sqrt{1.4398 \times 2\pi}}{(197.329)^2 \sqrt{0.693 \times 197.329 \times 0.333 \times 10^{-23}}}. \quad (39)$$

The expression (32) is some what similar to the VS relation mentioned previously but the difference is that in the present case the parameters and coefficients namely a , c , d , b_l are well defined. The value of the coefficient $a = 0.96$ and constant $d = -45.262$. The parameter c (35) depends on Z_D , A_D , Q_α and angular momentum partial wave l . Further, the parameter b_l is related to q_l (37) specifying the decay time for an α particle emitting with different angular momentum l .

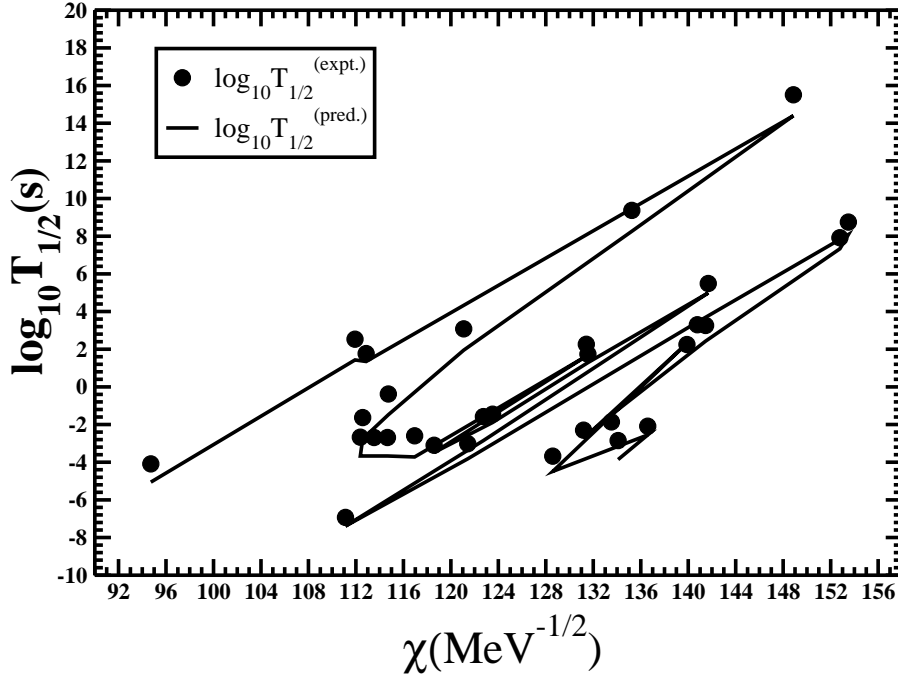


Fig. 2 Plot of decimal logarithm of half-lives $\log_{10}T_{1/2}^{(expt.)}$ from experiments (solid dots) [27, 28] and $\log_{10}T_{1/2}^{(pred.)}$ from calculation using (32) (solid line) as a function of χ in $l = 0$ state for α emitters with $Z = 52 - 118$.

3 Results and Discussions

We take the solvable potential described in Eqn.(16) and denote it as effective Coulomb-nuclear potential for the α +nucleus system and change only the steepness of the interior side of the barrier i.e. d_1 . With the Q_α value with us and the wave function $u_{nl}(r)$ at resonance, we calculate the half-life $T_{1/2}$ by using (10) and denote it by $T_{1/2}^{(calt.)}$.

For easy handling of the problem we condense the integral J given by (11) and write in terms of c_f , x_m and F_l^{ps} as mentioned in Eqn.(28). Our analysis show that for different nuclei the values of c_f comes out to be in the range 0.4 to 1.5 for $l = 0$. We assign the value of c_f i.e. $c_f = 0.8$ for $l = 0$ and $c_f = 0.6$ for $l > 0$. Furthermore, using this c_f , we estimate the values of $T_{1/2}$ by using the closed form expression (32) for the decimal logarithm of half-life and represent it as $\log_{10}T_{1/2}^{(pred.)} = \log T_{1/2}^{(pred.)} / 2.30258$. At last we determine the values of $T_{1/2}^{(pred.)}$ from the already found out $\log_{10}T_{1/2}^{(pred.)}$. We then compare the calculated results $T_{1/2}^{(calt.)}$ using (10), experimental results $T_{1/2}^{(expt.)}$ and predicted $T_{1/2}$ values i.e. $T_{1/2}^{(pred.)}$ using (32) and present systematically in Table 1 for $l = 0$

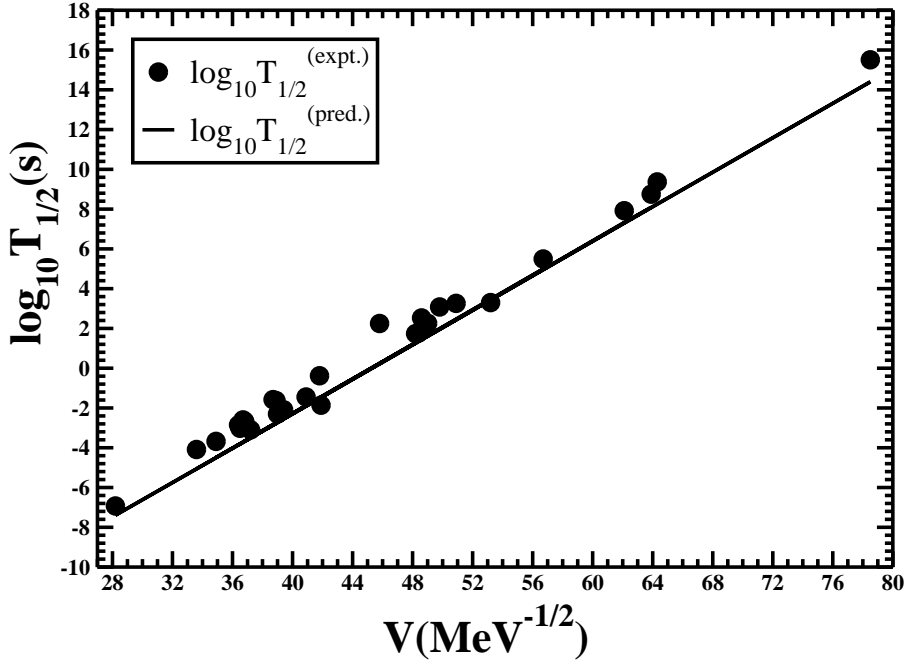


Fig. 3 Plot of decimal logarithm of half-lives $\log_{10} T_{1/2}^{(expt.)}$ from experiments (solid dots) [27, 28] and $\log_{10} T_{1/2}^{(pred.)}$ from calculation using (32) (solid line) as a function of $V = a\chi + c$ in $l = 0$ state for α emitters with $Z = 52 - 118$.

case. For a large assemblage of nuclei starting from $Z=52$ to 118, we write the $T_{1/2}$ values viz. $T_{1/2}^{(expt.)}$, $T_{1/2}^{(calt.)}$, $T_{1/2}^{(pred.)}$. Our findings reveal that we get a wide band of $T_{1/2}$ ranging from decimal logarithmic values of -7.39 s to 14.4 s. In the same way, we write the $T_{1/2}^{(expt.)}$, $T_{1/2}^{(calt.)}$, $T_{1/2}^{(pred.)}$ for $l > 0$ by taking $c_f = 0.6$ and present in Table 2. Our findings show that by changing the effective potential the c_f value is changing keeping in mind the results of Ref.[19].

Now to clarify the ambiguity on the non linearity of G N law for various α -emitters, we plot $\log_{10} T_{1/2}^{(expt.)}$ and $\log_{10} T_{1/2}^{(pred.)}$ as a function of the Coulomb parameter $\chi = Z_\alpha Z_D \sqrt{\frac{A_\alpha A_D}{(A_\alpha + A_D) Q_\alpha}}$ for $l = 0$ and present the plot in Fig. 2. Likewise, we also make a plot of $\log_{10} T_{1/2}^{(expt.)}$ and $\log_{10} T_{1/2}^{(pred.)}$ as a function of $V = a\chi + c$ used in (32) and present in Fig. 3. To our utter surprise, we find that the plot of $\log_{10} T_{1/2}^{(expt.)}$ and $\log_{10} T_{1/2}^{(pred.)}$ vs χ gives multiple straight lines whereas $\log_{10} T_{1/2}^{(expt.)}$ and $\log_{10} T_{1/2}^{(pred.)}$ vs $V = a\chi + c$ gives a single straight line. This clearly indicate that the our measured results have great accuracy.

Table 1 Comparison of experimental results of ground-state to ground state ($l = 0$) α -decay half-life $\log_{10}T_{1/2}^{(expt.)}$ in seconds with the calculated results $\log_{10}T_{1/2}^{(calt.)}$ in seconds using (10) and predicted values $\log_{10}T_{1/2}^{(pred.)}$ in seconds using (32). The experimental data of α -decay half-life and Q_α value in MeV are obtained from Ref.[27,28]. The value of the parameter d_2 is kept constant throughout i.e. $d_2 = 2$.

Nucleus	d_1	$Q_\alpha^{expt.}$ (MeV)	$\log_{10}T_{1/2}^{expt.}$ (s)	$\log_{10}T_{1/2}^{calt.}$ (s)	$\log_{10}T_{1/2}^{pred.}$ (s)
$^{106}_{52}$	6.41600	4.290	-4.09	-4.47	-5.06
$^{112}_{54}$	4.77400	3.330	2.53	1.80	1.43
$^{114}_{56}$	4.72599	3.534	1.77	1.67	1.35
$^{146}_{62}$	4.90799	2.528	15.51	15.15	14.4
$^{148}_{64}$	4.94400	3.271	9.37	10.05	8.26
$^{150}_{66}$	5.02799	4.3510	3.08	3.80	1.95
$^{154}_{70}$	5.04400	5.474	-0.38	-0.40	-1.48
$^{156}_{72}$	6.50199	6.028	-1.63	-0.67	-2.76
$^{162}_{76}$	5.03600	6.767	-2.67	-2.34	-3.67
$^{168}_{78}$	5.06200	6.999	-2.69	-2.94	-3.67
$^{174}_{80}$	5.09399	7.233	-2.69	-2.11	-3.67
$^{190}_{84}$	5.21400	7.699	-2.60	-2.61	-3.72
$^{218}_{84}$	5.41800	6.115	2.26	4.34	1.63
$^{194}_{86}$	5.18000	7.862	-3.10	-1.58	-3.48
$^{218}_{86}$	4.22399	7.263	-1.45	-1.37	-1.89
$^{220}_{86}$	4.12200	6.405	1.74	1.97	1.29
$^{224}_{88}$	5.19999	5.789	5.49	7.99	4.99
$^{216}_{90}$	4.07800	8.071	-1.58	-2.37	-2.85
$^{218}_{90}$	4.39800	9.849	-6.93	-7.06	-7.39
$^{224}_{92}$	5.45800	8.633	-3.02	-1.92	-3.82
$^{232}_{94}$	6.48600	6.716	3.30	6.44	3.44
$^{244}_{96}$	5.03000	5.902	8.75	11.13	8.09
$^{252}_{98}$	5.07000	6.217	7.92	10.63	7.33
$^{250}_{100}$	5.147990	7.556	3.26	5.27	2.43
$^{258}_{104}$	4.05800	9.190	-1.86	-0.26	-1.48
$^{260}_{106}$	5.34000	9.901	-2.30	-0.68	-2.74
$^{270}_{108}$	6.58800	9.050	2.25	3.01	2.31
$^{270}_{110}$	4.18599	11.120	-3.68	-3.05	-4.49
$^{290}_{116}$	4.07999	10.990	-2.09	-0.77	-2.55
$^{294}_{118}$	4.15000	11.810	-2.85	-2.22	-3.86

Table 2 Logarithm of calculated α -decay half-lives $\log_{10}T_{1/2}^{(calc.)}$ in seconds using (10), logarithm of predicted α -decay half-lives $\log_{10}T_{1/2}^{(pred.)}$ in seconds using (32) with parameter fixed $c_f = 0.8$ for $l = 0$ and $c_f = 0.6$ for $l > 0$. The experimental results of half-lives $\log_{10}T_{1/2}^{(expt.)}$ in seconds [28]. The α -decay energies Q_α in MeV are taken from atomic mass table [27]. The value of the parameter d_2 is kept constant throughout i.e. $d_2 = 2$.

Nucleus	d_1	$Q_\alpha^{expt.}$ (MeV)	l	$\log_{10}T_{1/2}^{expt.}$ (s)	$\log_{10}T_{1/2}^{calc.}$ (s)	$\log_{10}T_{1/2}^{pred.}$ (s)
$^{105}_{52}$	4.88800	4.889	0	-6.19	-7.42	-7.45
$^{109}_{54}$	6.32800	4.217	2	-1.52	-2.57	-3.83
$^{147}_{62}$	4.89599	2.331	0	18.52	17.05	14.50
$^{157}_{70}$	4.96400	4.621	0	1.58	4.32	2.60
$^{159}_{74}$	5.04800	6.450	0	-2.08	-2.74	-3.43
$^{165}_{74}$	4.93400	5.029	2	0.70	4.47	2.24
$^{161}_{76}$	5.06400	7.066	0	-3.19	-3.93	-4.61
$^{167}_{78}$	5.06599	7.160	0	-3.09	-2.58	-4.17
$^{171}_{78}$	5.05599	6.610	0	-1.34	-1.29	-2.40
$^{175}_{80}$	5.07399	7.043	0	-1.97	-1.62	-3.08
$^{187}_{84}$	6.65999	7.979	2	-2.85	-2.37	-4.78
$^{189}_{84}$	5.19199	7.701	2	-2.42	-2.01	-3.99
$^{209}_{84}$	5.07600	4.979	2	9.50	8.93	7.28
$^{211}_{84}$	4.28200	7.595	5	-0.28	-3.07	-3.69
$^{209}_{86}$	6.54600	6.156	0	3.23	5.46	2.50
$^{211}_{86}$	5.14599	5.965	2	4.72	6.18	3.06
$^{213}_{86}$	4.29400	8.244	5	-1.70	-3.61	-4.86
$^{221}_{86}$	4.09399	6.148	2	3.18	4.27	2.10
$^{203}_{88}$	5.19400	7.730	0	-1.44	-0.52	-2.41
$^{217}_{90}$	4.30800	9.433	5	-3.60	-5.40	-6.54
$^{221}_{90}$	4.24200	8.628	2	-2.77	-3.05	-4.79
$^{225}_{90}$	5.28600	6.920	2	2.72	3.64	6.59
$^{219}_{92}$	4.30600	9.940	5	-4.25	-6.59	-7.06
$^{235}_{92}$	4.935990	4.678	1	16.34	16.26	13.3
$^{239}_{94}$	4.97000	5.245	3	11.88	14.17	10.6
$^{243}_{96}$	5.05399	6.169	2	8.96	9.95	6.45
$^{245}_{96}$	4.99200	5.622	2	11.42	12.91	9.35
$^{247}_{96}$	4.97599	5.3540	1	14.69	14.64	10.8
$^{249}_{98}$	5.04400	6.296	1	11.4	9.18	6.60
$^{251}_{98}$	5.05399	6.177	5	10.45	10.28	7.51
$^{251}_{100}$	6.52000	7.425	1	4.28	5.87	2.54
$^{253}_{100}$	5.13600	7.199	5	5.41	6.47	3.72
$^{255}_{100}$	5.17399	7.240	4	4.87	6.35	3.36
$^{257}_{100}$	5.13800	6.864	2	6.95	8.10	4.86
$^{267}_{110}$	5.53399	11.780	0	-5.0	-3.85	-5.85
$^{273}_{110}$	4.26599	11.370	0	-3.61	-3.95	-5.07
$^{277}_{112}$	6.97000	11.620	0	-3.00	-2.73	-5.05
$^{291}_{116}$	4.07200	10.8900	0	-1.55	-1.19	-2.32

4 Summary and Conclusion

By using the regular Coulomb function, resonant wave function and the difference in potentials a general formula is being put forth for the calculation of α -decay width. The α +nucleus potential generated by using relativistic mean field theory is closely reproduced by special expressions of the potential and finally we zero in to a closed formula for the logarithm of half-life as a function of Q -values of various decaying nuclei of different masses and charges. This derived formula is impeccable in finding the logarithm of half-lives. The closed formula for the logarithm of half-life favorably explains the half-lives ranging from 10^{-6} s to 10^{22} y. Also this closed form expression curtains the dilemma over nonlinearity as it fairly reproduces the rectilinear alignment of the logarithm of the experimental decay half-lives as a function of the Viola-Seaborg parameter. With the updated VS rule with us, we only need the Q_α value, mass number A_D and charge Z_D to predict the α -decay half-life and there by future work concerning half-lives of all types of nuclei can be foresighted.

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